to the photometer, while part would reach the photometer directly. The author hopes to have the opportunity of further considering this application.

In conclusion, he has great pleasure in acknowledging his indebtedness to Dr. Glazebrook, the Director of the Laboratory, for his kindness in reading and revising the paper, and also to Mr. F. J. Selby for similar help. Many of the measurements were taken by Mr. T. Smith, the author's successor at the laboratory, and to him also the author desires to express his thanks.

The Flight of a Rifled Projectile in Air.

By Prof. J. B. Henderson, D.Sc., Royal Naval College, Greenwich.

(Communicated by W. Burnside, F.R.S. Received February 9,—Read February 25, 1909.)

The stability of a moving projectile has been treated by Sir George Greenhill as a practical application of a problem in hydrodynamics which he had previously solved, namely, the stability of a rotating spheroid moving slowly through a liquid.* This hydrodynamical illustration, however, though in itself interesting, gives little assistance in the numerical treatment of the problem which is here discussed—the stability of a projectile after it is launched on its trajectory. The shot is then moving faster than a wave of compression in air; for this reason, and on account of the eddying motion generally, the shot cannot be linked to the air in that closed kinematic chain which is assumed in all problems in hydrodynamics, and in which the velocity of every particle of fluid depends only on the velocity of the solid, the two varying together in a perfectly definite manner. Only the air at a very short distance from the projectile can directly affect the motion of the latter, and in the following pages the problem is treated simply as that of a moving rotating body meeting with certain resistances.

This method of treating the problem seems natural and self-evident. It has been used by the writer in lecturing to naval gunnery lieutenants during the last four years, and by Mr. A. Mallock, F.R.S., in a paper on "Ranges and Behaviour of Rifled Projectiles in Air,"† but, so far as the writer is aware, it has never before been carried to its ultimate conclusion—the synthesis of a trajectory in all its details, the initial conditions and the laws

* See 'The Engineer,' November—December, 1907.
† 'Roy. Soc. Proc.,' June 24, 1907.

of resistance alone being assumed. Incidentally the causes of both horizontal and vertical "drift" are made manifest.

The projectile having left the gun with a certain initial velocity and a sufficient spin about its axis of figure to ensure stability about that axis, the direction of motion being inclined to the axis of figure, let us try to trace step by step the path followed by the centre of gravity of the shot and that traced out in space by the axis of figure.

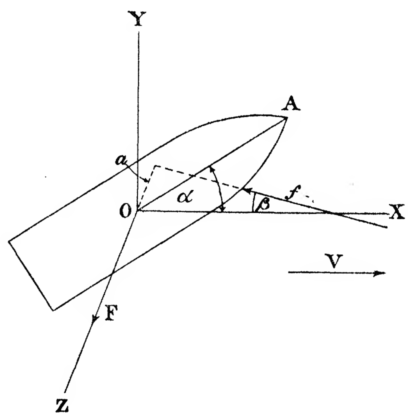

FIG. 1.

Let OX be the direction of motion of the centre of gravity O (fig. 1), the axis of figure OA being deflected through a small angle α from OX; and suppose that, at the instant, OA is in the plane XY. The air resistance is then represented by a force f which does not pass through the centre of gravity, but in modern projectiles intersects the axis in front of the centre of gravity. We thus have acting on the projectile a "tilting" couple fa, a force $f \cos \beta$ decelerating the speed and a force $f \sin \beta$ generating momentum perpendicular to OX in the plane AOX. The couple causes OA to precess round OX, the velocity of precession perpendicular to AOX being $\frac{\partial \psi}{\partial t} = \frac{fa}{\mathrm{I}\varpi}$ (where I is the moment of inertia of the projectile about its axis of figure and ϖ its angular velocity of spin). Whence the period of precession round OA is

$$\mathrm{T} = 2\pi \sin \alpha \bigg/ \frac{\partial \psi}{\partial t} = \frac{2\pi \mathrm{I}\varpi \sin \alpha}{fa}.$$

The component force $f \sin \beta$ normal to OX gives a curvature to OX in the plane AOX, and since this component is always in the plane AOX and precesses with the shot round OX, the resulting curvature is constantly changing in direction and causes OX to assume a helical form.

The curvature of the path is given by

$$\frac{d\theta}{dt}=\frac{f\sin\beta}{m\mathrm{V}} \quad \text{and} \quad \frac{ds}{dt}=\mathrm{V}, \quad \text{whence} \quad \frac{d\theta}{ds}=\frac{f\sin\beta}{m\mathrm{V}^2},$$

where m is the mass and V the velocity of the projectile.

In practice, both α and β will be small angles, and the sines may be replaced by the angles themselves. Let us first of all assume the force of gravity to be annulled.

In order to represent the changes in OA and OX by means of a diagram, since it is directions in space we are considering, let us represent angles by their traces on a spherical surface. Let a huge sphere be circumscribed round the projectile of radius so large that the dimensions of the trajectory are negligible compared with it, let us say the celestial sphere; and let the diagram represent the paths traced out on the celestial sphere by the prolongation of OA and OX. To simplify the drawing of the diagram let us divide the period of precession of OA into 12 equal parts, and consider OX the direction of motion to be fixed during each interval, and at the end of the interval add an equivalent change of direction in OX. This change of direction would be given by $\theta=\frac{f\sin\beta}{m\mathrm{V}}\cdot\frac{\mathrm{T}}{12}$, where T is the period of precession, and since α and β vary together we must, for the purposes of a diagram, make some assumption as to the connection between them. Let us assume that β is simply proportional to α. Then θ is simply proportional to α if T is constant. But $\mathrm{T}=2\pi\mathrm{I}\varpi\sin\alpha/fa$, and we may consider a proportional to $\sin\alpha$, and therefore T is constant, or at least independent of α.

In fig. 2, A_0 is the intersection of OA with the celestial sphere at a particular instant, and X_0 the simultaneous intersection of OX with the sphere. The centre of the sphere is at O (not shown in the diagram), and the projectile is supposed to be approaching the observer along OX_0 perpendicular to the plane of the diagram. In one-twelfth of a period OA precesses round OX from OA_0 to OA_1, and simultaneously OX moves towards OA from OX_0 to OX_1, $\mathrm{X}_0\mathrm{X}_1$ being θ and being drawn in the mean direction of XA during the interval. Similarly the successive positions $\mathrm{A}_2, \mathrm{A}_3\ldots$ $\mathrm{X}_2, \mathrm{X}_3\ldots$ are obtained. It is seen that both A and X have spiral traces on the celestial sphere, and that the angle AOX steadily decreases; also that, if the initial deflection of the axis of the shot is upwards, as supposed in the diagram, the direction of motion OX drifts to the right and simultaneously rises above the original direction OX_0. From this diagram and the forward velocity V it is easy to draw a diagram of the spiral surface traced out by the axis of the shot. This is shown in fig. 3 with the scale of lateral deviation greatly magnified. It

will be seen from the elevation that instead of the trajectory remaining along the original horizontal line OX_0 it rises above this line, and from the plan it will be observed that it deviates considerably to the right. We shall see later that these effects have been greatly magnified in figs. 2 and 3, but it is important to notice that they are present in every trajectory, whether gravity

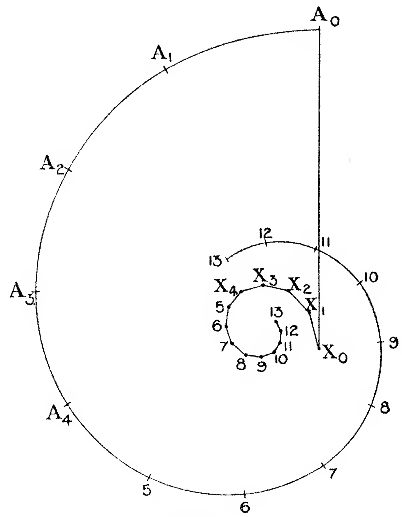

FIG. 2.

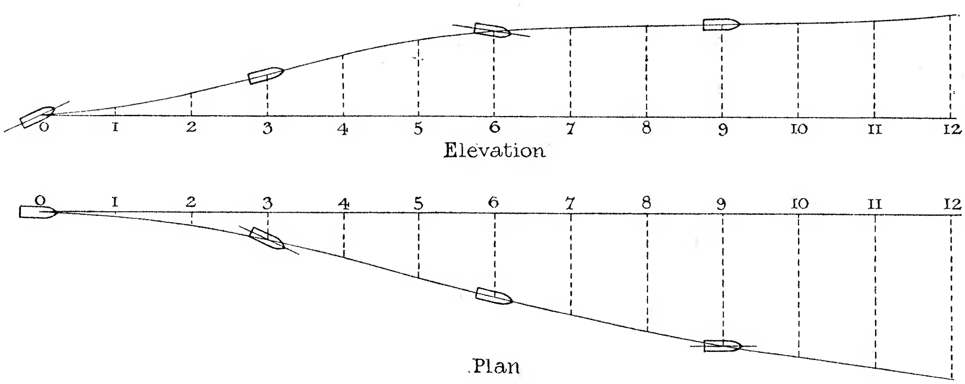

FIG. 3.

is neglected or not. One effect of magnifying the force $f \sin \beta$ is that the spiral motion almost disappears in one turn of the spiral. This point is discussed later.

The question may be asked: Have we any experimental evidence of this helical path? No accurate experiments have been made on the subject so far as the writer is aware. One hears of actual spiral paths being observed by persons stationed behind the gun, but the evidence on this point is very

conflicting. Undoubtedly the projectile does occasionally leave the gun with its axis considerably inclined to the direction of motion, and in such a case the spiral motion will be exaggerated; but this is the exceptional case, and we shall see later that the dimensions of the spiral must, as a rule, be such as would be invisible to the eye by the time the shot first becomes visible—at, say, 100 yards distance. The shot will as a rule leave the gun with its axis slightly inclined to the direction of motion. This inclination may be produced by the rush of gases past the shot at the moment of leaving, or by a parting kick from the gun in a radial direction just as the base of the shell is leaving the muzzle. Such a kick may arise from "whip" in the gun, or if a field gun is at all elevated, it will arise from any recoil of the gun carriage as a whole, which, being in the horizontal plane, is inclined to the direction of motion of the shot. In the interval of time between the point of the shell and the base of the shell leaving the gun, the gun must gain velocity of recoil, which it imparts to the base; hence the base gets a knock upwards and the axis of the shot immediately inclines to the left in consequence.

The direct evidence available which would throw light on this helical motion is confined to the distribution of shot marks on a target at short range, from a gun carefully sighted on the centre of the target. This evidence would not be conclusive unless artificial arrangements were introduced for giving the shot an initial deflection to opposite sides, say by arranging the recoil horizontally and then firing at two targets, one at 10° elevation and the other at 10° depression, and then comparing the targets. The radius of the helix must be of the order of 1 inch [but see addendum], consequently the holes pierced by a large shot in a series of equally spaced screens could not throw any light on the subject, owing to the difficulty of accurately surveying the holes. If we consider the period of revolution in the spiral, that is the period of precession, the problem seems more hopeful. The shot in its path towards the target has at each moment maximum and minimum air compression on opposite sides, and these positions of maximum and minimum pressure are presented alternately to an observer stationed down the range at some distance laterally from the target; and since a compression is a seat of generation of atmospheric waves, the waves which reach the observer must have a slow pulsation of the periodicity equal to that of the precession in the spiral. Whether these waves are audible or not the writer cannot say, but a sensitive manometric flame or a microphone diaphragm would probably disclose the period. They ought to be heard by an observer situated down the range in the interval between seeing the flash and hearing the report. It may be that the

shot is not large enough to throw the sound shadow on which the periodic pulsations would depend.

A projectile having left the gun with its axis deflected from the direction of motion, let us now study the forces which reduce this deflection, and which tend in general to keep the axis of the projectile tangential to the trajectory. These are the friction couple and the force already considered—the component of f normal to the direction of motion. The former acts directly in altering the direction of the axis of the projectile so as to reduce the deviation; while the latter affects the direction of motion of the centre of gravity in such a manner as also to produce the same effect. We shall consider the friction couple first.

Since the air pressure is unsymmetrically distributed round the projectile, the friction forces tending to stop the rotation will also be unsymmetrically distributed round the axis; for, if we consider the force f as being due to an excess cushion of air on one side of the projectile, the rotation of the shot will introduce an excess of friction in the peripheral direction in the neighbourhood of the excess cushion of air. Thus we shall have a friction force q perpendicular to the plane AOX. Or, considering a section of the projectile by a plane containing OX and perpendicular to the plane AOX, we get a figure somewhat like fig. 4. There will be a dense wedge-shaped cushion of air M

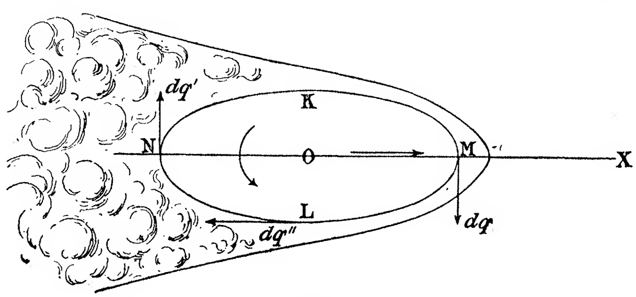

Fig. 4.

on the advancing side, and a tail race N of eddies in the wake. The rotation of the projectile is in the direction of the arrow, and this rotation imparts momentum to the cushion M and to the wake N in the peripheral direction; hence we have the equivalent of two forces, dq and dq', acting on the projectile at M and N. The relative velocities of the air and the projectile at K and L are different, hence we have an additional tangential force at L represented by dq''.

If the forward velocity of the projectile were small as in a golf ball, this extra retardation of the air on the side L would cause an accumulation of air on that side, and the cushion M would be moved slightly round on the

side L, the stream lines being modified accordingly. There would then be a component pressure perpendicular to the plane AOX. In golf the spin about a vertical axis on a " sliced " or " pulled " ball causes this displacement of the cushion of air on the front of the ball, and a drift to right or left is the result, the rate of drift increasing as the velocity falls off.

In a projectile, however, the forward velocity is very great compared with the peripheral velocity due to any component spin about a vertical axis, and the rate at which the air in the cushion M is being renewed by the eddies effectually prevents any great deflection of the cushion towards the side L; the difference of the relative velocities of air and metal at K and L is negligible, and the force dq'' is zero. Hence the resultant resistance f is approximately in the plane AOX as assumed in fig. 1, and the drifts of a projectile and of a golf ball are due to different causes. The effect of the forces dq and dq' over the whole projectile might be represented by two forces q and q' in fig. 5 perpendicular to the plane AOX. These are

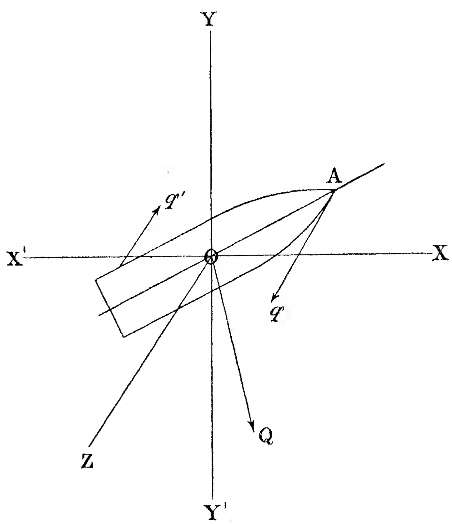

Fig. 5.

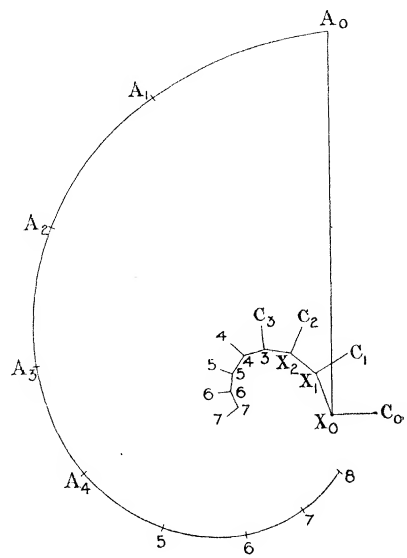

Fig. 6.

equivalent to a force $q-q'$ through the centre of gravity and a couple having OQ as axis, OQ being in the plane AOX, or, in this figure, in the plane XY. The axis OA will therefore precess under the action of the friction couple so as to tend to coincide with OQ; and it will continue to do so until OX is reached—that is, until the axis OA is tangential to the trajectory, in which position the friction couple q vanishes, the friction then being symmetrical about OA.

The influence of the friction couple is shown in fig. 6, but not to scale. The f couple gives to OA a precession perpendicular to the plane AOX, and the q couple gives a precession in the plane AOX towards X. The forces of the f couple are great compared with the forces of the q couple, but the arm of the q couple is great compared with the arm of the f couple. If we assume the f couple to be 10 times the q couple the centre about which A is turning is C_0, where C_0X_0 is $\frac{1}{10}A_0X_0$. The arc A_0A_1 is drawn with C_0 as centre. The centripetal component $f \sin\beta$ will also be modified in direction by the small friction force $q-q'$ perpendicular to it. If the arms of the f and q couples were equal the resultant direction would be C_0A_0 in the initial position, hence X_0X_1 would be parallel to the mean direction of C_0A during the first interval; but whether we draw X_0X_1 parallel to the mean direction of C_0A or to the mean direction of X_0A makes little difference to the diagram. The friction couple modifies the trace of A very considerably, as will be seen by comparing figs. 2 and 6, the rate of diminution of the obliquity being considerably increased. It will also be noticed that the drift is still to the right and upwards.

Let us now try to get some idea of the magnitude of this helical motion by considering an actual example; let—

p = pitch of helix.
I = moment of inertia of projectile about axis.
ϖ = angular velocity of projectile about axis.
r = pitch of rifling.
a = arm of f couple (fig. 1).
α = inclination of axis to direction of motion.
β = inclination of f to direction of motion.
T = period of precession in helix.
V = velocity of projectile.

Then we have $$\mathrm{T} = \frac{p}{\mathrm{V}}, \qquad \varpi = \frac{2\pi}{r/\mathrm{V}} = \frac{2\pi\mathrm{V}}{r}.$$

Precessional velocity of axis OA perpendicular to the plane AOX is

$$\frac{\partial\psi}{\partial t} = \frac{2\pi}{\mathrm{T}}\sin\alpha = \frac{2\pi\mathrm{V}}{p}\sin\alpha.$$

Tilting couple $= fa = \mathrm{I}\varpi\dfrac{\partial\psi}{\partial t} = \dfrac{2\pi\mathrm{I}\varpi}{\mathrm{T}}\sin\alpha = \dfrac{4\pi^2\mathrm{IV}}{\mathrm{T}r}\sin\alpha$, therefore

$$\mathrm{T} = \frac{4\pi^2\mathrm{IV}}{fr}\cdot\frac{\sin\alpha}{a}.$$

Now a depends on α, the connection between them depending on the shape of the ogival head. If we limit the problem to small deviations we

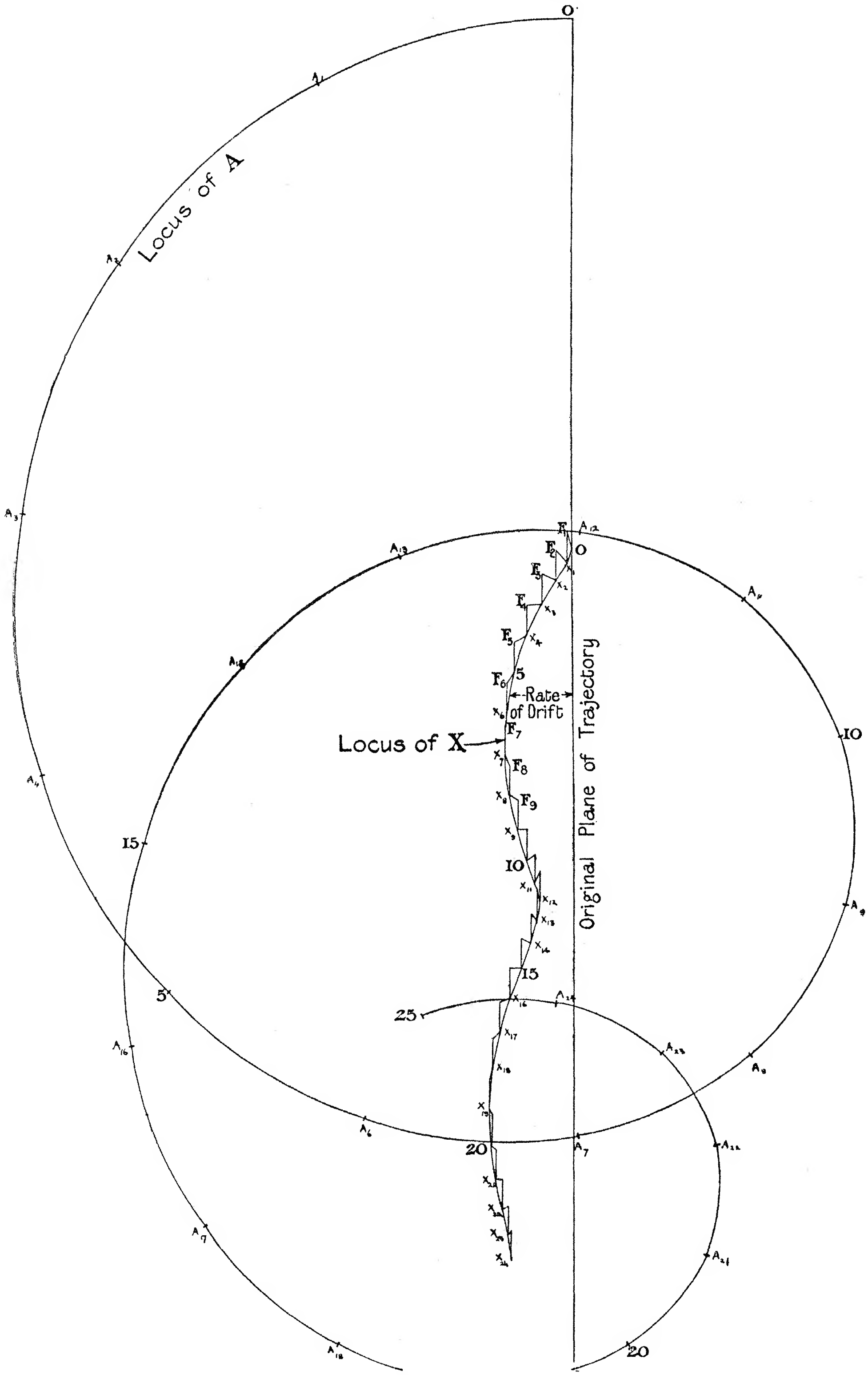
Locus of A
Locus of X
Rate of Drift
Original Plane of Trajectory

Fig. 7.

may write the angle for the sine of the angle, and by considering the shape of the head we see that β will at first increase much faster than α, since the pressure on one side of the ogive is increasing and on the other side decreasing as α increases. Taking β to be about 3α and α as one degree in a 12-inch shell, a will be of the order of 1 inch. The square of the radius of gyration may be taken as one-half the square of the radius of the projectile. The resistance f at a velocity of 2500 feet per second is approximately four times the weight of the projectile, and r is 45 feet.

$$\text{Thus} \qquad \mathrm{T} = \frac{4\pi^2 m \times 0{\cdot}125 \text{ ft.}^2 \times 2500 \text{ ft./sec.}}{4 \times m \times 32 \text{ ft./sec.}^2 \times 45 \text{ ft.}} \times \frac{\frac{1}{60}}{\frac{1}{12}\text{ ft.}}$$
$$= 0{\cdot}44 \text{ second.}$$

There may be an error in the estimation of a of 100 per cent., so all we can say is that the period is of the order of 1 second.

Influence of Gravity on the Trajectory.

We are now in a position to discuss the effects of gravity on the trajectory. The weight may be resolved into two components, one along and the other perpendicular to the direction of motion. The first varies the magnitude of the velocity but not its direction, while the second varies the direction of motion. We are only concerned at present with the second component. The vertical plane of motion oscillates backwards and forwards in azimuth due to the helical motion, and in this plane, in every position it occupies, the normal component of the weight gives to the direction of motion a curvature downwards. Let us trace out on the celestial sphere the changes of direction of both the axis of the projectile and the direction of motion due to the weight. In order not to complicate the diagram let us first of all neglect the friction couple, and let us choose the scales of the various magnitudes so as to make a clear diagram, but not to represent the physical facts. These will be considered later. In fig. 7, A_0 and X_0 are the initial intersections as before. During the first twelfth of a period A precesses from A_0 to A_1, and due to the component $f \sin\beta$, if it acted alone, X_0 would move to F_1, but since the weight has also been acting during the interval it lowers X by the displacement F_1X_1. Hence, due to the combined action of f and W, X moves from X_0 to X_1 during the first twelfth of a period. During the second twelfth A turns about X_1 from A_1 to A_2, and X moves simultaneously from X_1 to X_2, and so on.

Both the motion of A and the radial motion of X are proportional to the obliquity AOX, but the vertical motion of X is independent of the obliquity. Hence the vertical displacements F_1X_1, F_2X_2, etc., are all equal, while the

radial motions X_0F_1, X_1F_2,... etc., are proportional respectively to A_0A_1, A_1A_2, etc., that is, to A_0OX_0, A_1OX_1, etc. The traces of A and X are sketched in fig. 7 for two whole periods of the helical motion. It will be noticed that the obliquity AOX gradually diminishes and that the direction of motion bears to the right. The horizontal ordinate of the locus of X is proportional to the rate of horizontal drift, and it would be easy to plot from this curve a curve of rate of drift as a function of time, and then by integrating it to get the linear drift at any moment. It will be noticed that the drift as drawn is always to the right, but suppose as a starting point for our curve we take, say, A_6, that is, we assume that the axis of the projectile is initially inclined downwards from the direction of motion, then the original plane of the trajectory is the vertical through X_6 and the drift is then to the left for almost the whole of the first complete period. We see, therefore, that it must be impossible to obtain an analytical formula for drift which would apply to all guns or to all elevations of the same gun, since, as we have seen, the initial deviation of the axis of the shot is likely to vary with different guns, and with the same gun at different elevations. The natural elevation of a field gun would cause an initial deflection in the direction corresponding to A_{10} if the carriage recoiled horizontally, and the initial plane of the trajectory would be a vertical through X_{10}. Hence we should expect the shot to show drift to the left for short ranges and to the right for long ranges if the elevation were kept constant. The elevation, however, is varied with the range, and in modern guns the recoil is parallel to the axis of the gun; hence the drift to the left at short ranges, which has been noticed in certain guns, must be caused by an initial deflection of the axis of the projectile to the left due to "whip" in the gun in the vertical plane, or to the rush of gases past the shot.

Numerical Example.

In order to get some idea of the relative magnitudes of the displacements of the direction of motion due to the weight and the normal component of the resistance, let us consider a 12″ shell moving at 2500 feet per second with an obliquity of axis of one degree, and suppose that the corresponding obliquity of the resistance f is three degrees. The deceleration at this speed is about $4g$. The period of precession is, say, 0·48 second, and the intervals under consideration are 0·04 second. The change of direction during 0·04 second due to the weight is given by

$$F_1X_1 = \frac{gt}{V} = \frac{32 \times 0{\cdot}04}{2500} \text{ radian} = 1{\cdot}8 \text{ minutes of arc.}$$

The normal acceleration due to $f \sin \beta$ is approximately β times the deceleration, or $\frac{1}{20} \times 4g = \frac{1}{5}g$. Hence

$$X_0F = \tfrac{1}{5}F_1X_1 = 0{\cdot}36 \text{ minute of arc.}$$

Hence, if we take the distances X_0F_1, F_1X_1, and A_0X_0 in the ratio 1 : 5 : 180, we shall have a diagram approximating to an actual case. Such a diagram is drawn in fig. 8, but the friction couple has been omitted in order to show more clearly the effect of friction by a separate diagram. Assuming the friction couple to be one-tenth of the direct resistance or tilting couple, and redrawing the diagram as in fig. 6, we get fig. 9. This figure is important as including all the effects and showing a real trajectory approximately to scale so far as the angular deviations of the axis and the direction of motion are concerned. It will be noticed by comparing figs. 8 and 9 that the friction couple is the principal agent in reducing the deviation of the axis of the shot from the direction of motion; that the effect of the normal component of the resistance is negligible in comparison, but that this normal component of the resistance is the active agent in producing drift.

In fig. 9 four complete periods are shown, and the deviation has been reduced from one degree to almost zero. In order to study the deviation in the case when the axis is initially coincident with the direction of motion fig. 10 is drawn, maintaining the same relative magnitudes as were used in fig. 9, but magnifying the scale of the whole diagram five times, so that a single loop at the foot of fig. 9 represents fig. 10. It is seen that although the axis and direction of motion are initially coincident they are never again coincident, and that the axis keeps always to the right of the direction of motion. In such a case drift to the left is of course impossible. Since the $f \sin \beta$ effects are negligible, the vertical scale becomes a scale of time, and the locus of X becomes a curve of rate of drift. In order to show the shape of this curve the ordinates have been magnified 45 times, and the new curve so obtained is called a curve of rate of drift. The area of this curve if integrated gives the linear drift.

Fig. 11 is drawn to show the effect of doubling the friction couple. This effect, as will be seen by comparing figs. 10 and 11, is very slight—if the initial deviation is zero—during the short interval of time considered in the diagram.

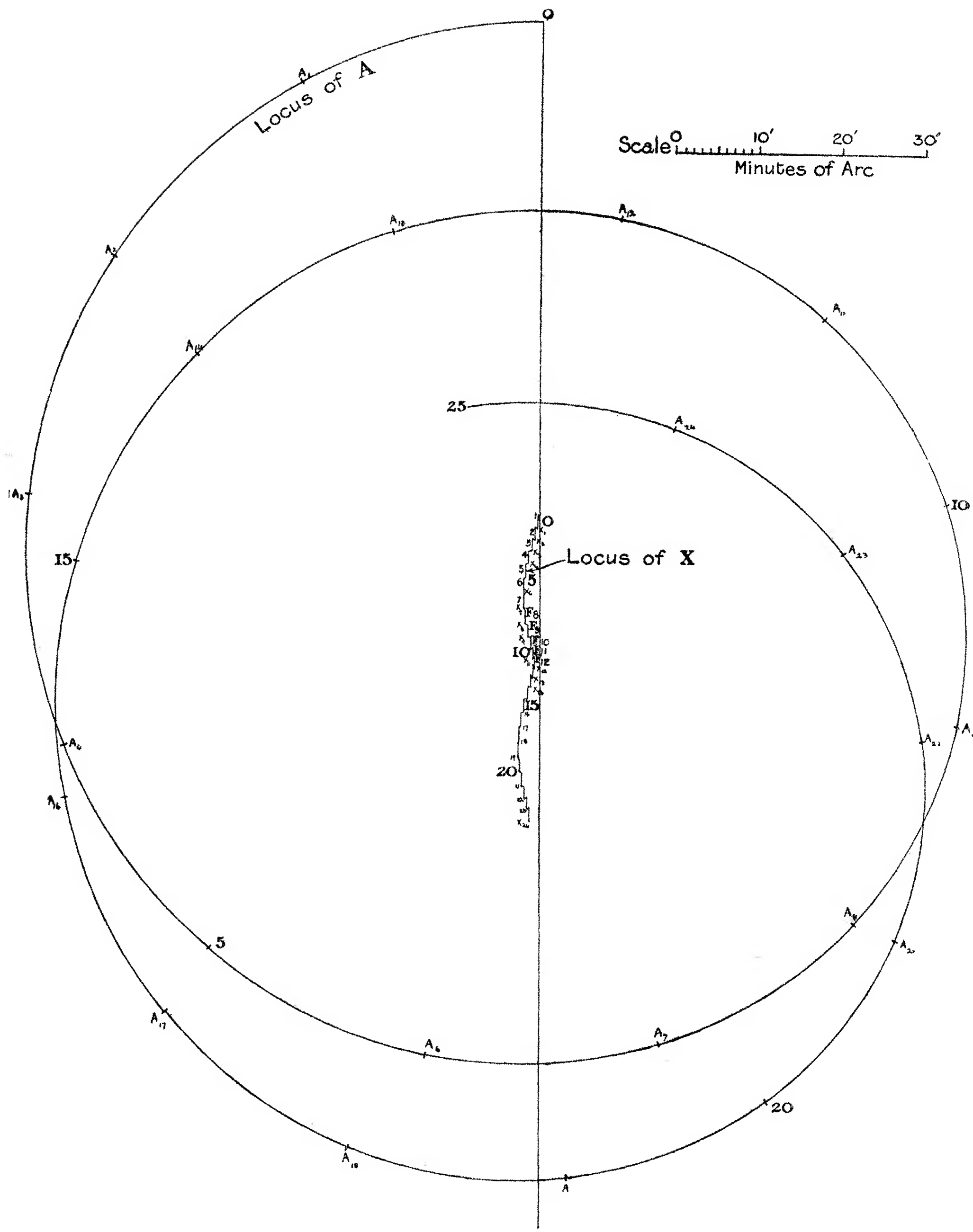

Fig. 8.

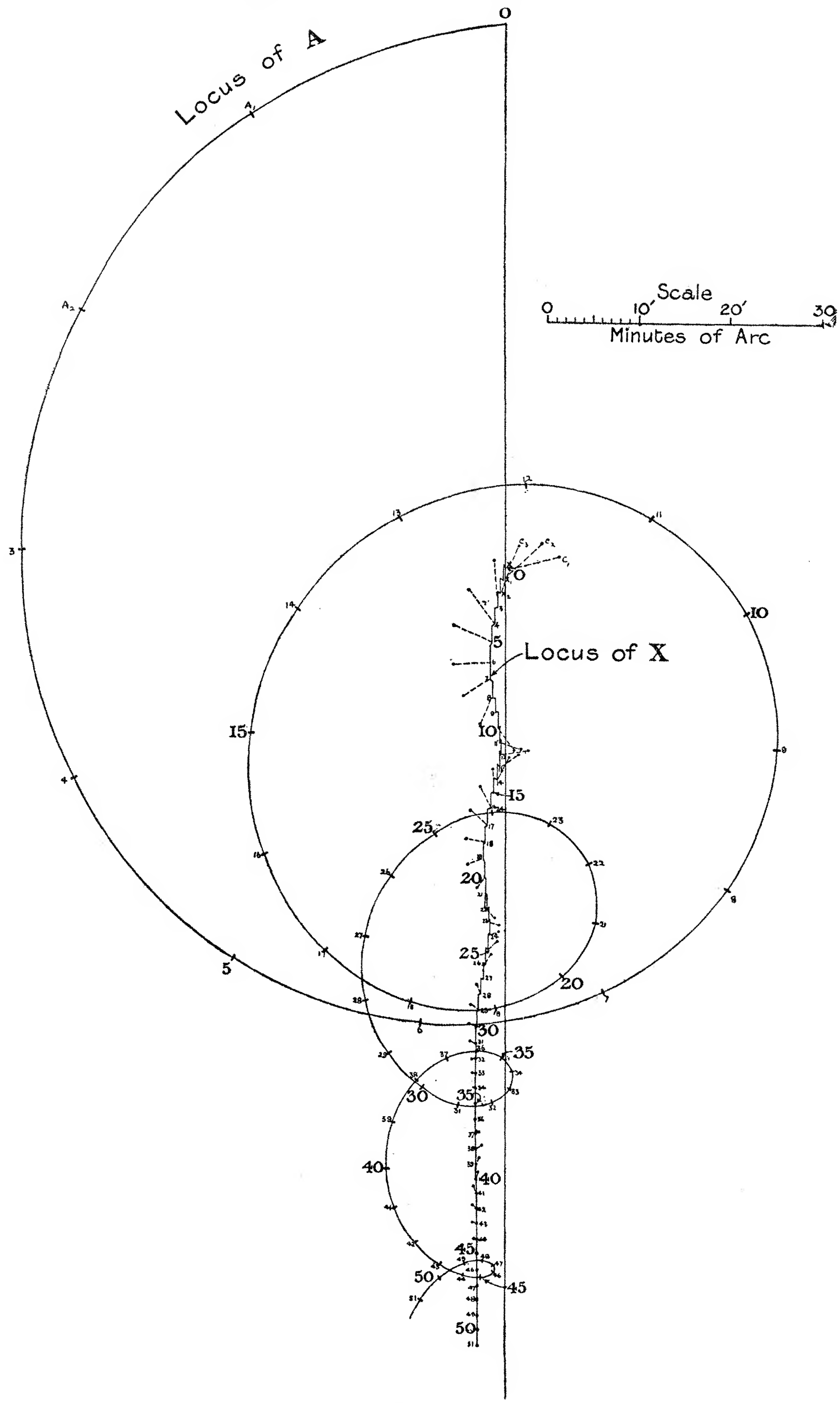

FIG. 9.

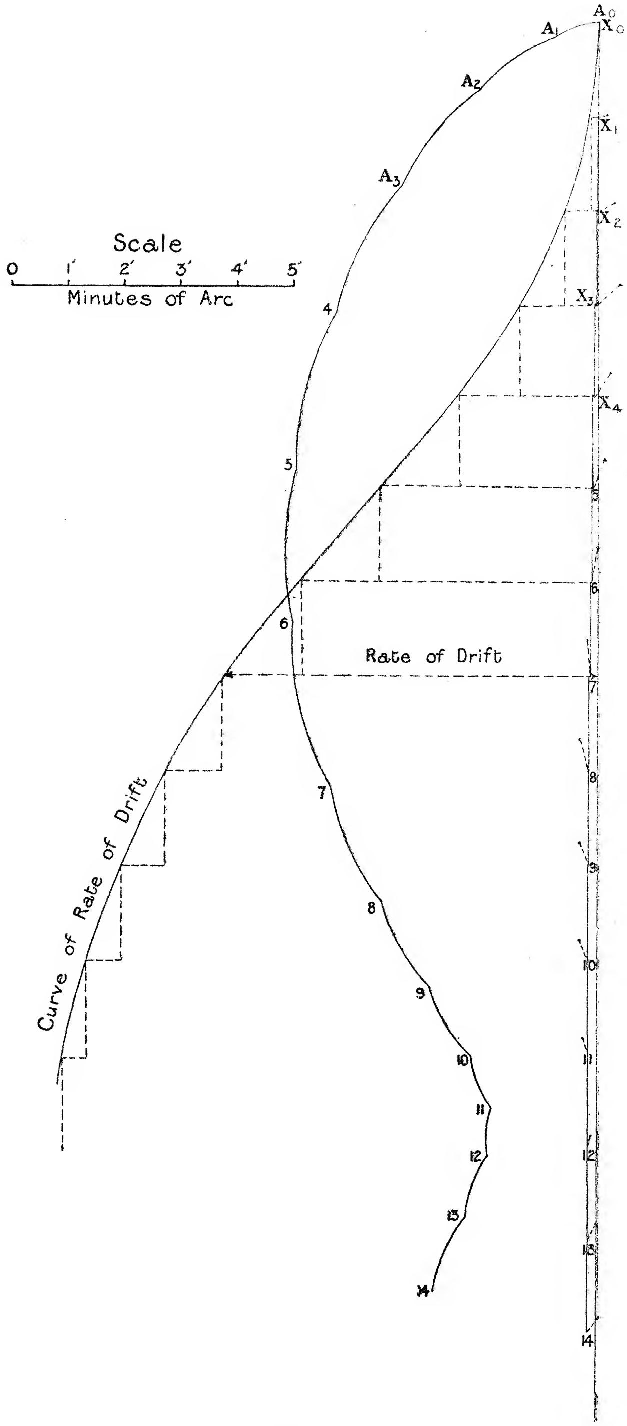
A0
X0
A1
A2
A3
X1
X2
X3
X4
Scale
0 1' 2' 3' 4' 5'
Minutes of Arc
4
5
6
7
8
9
10
11
12
13
14
Rate of Drift
Curve of Rate of Drift

FIG. 10.

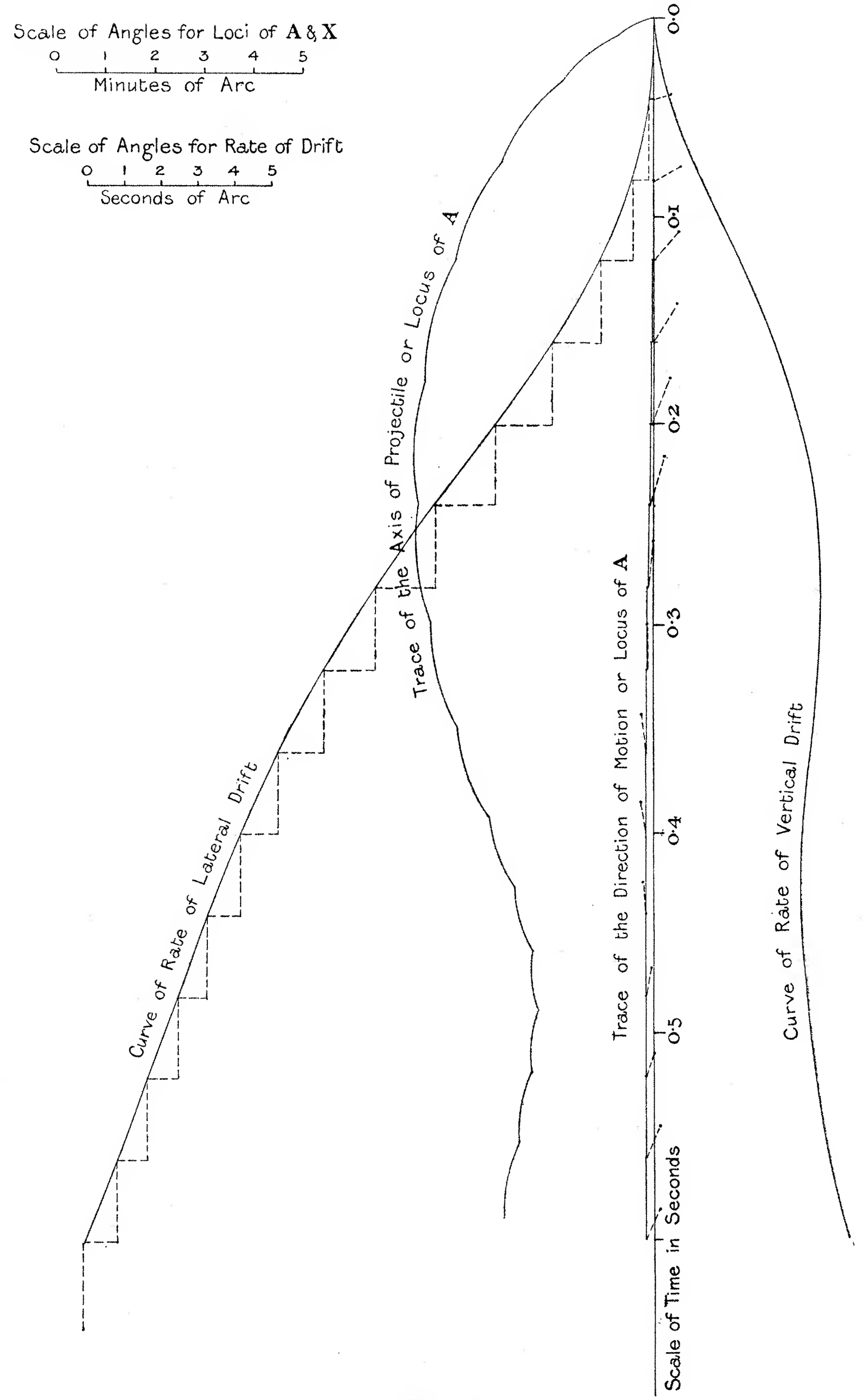
Scale of Angles for Loci of A & X
0 1 2 3 4 5
Minutes of Arc
Scale of Angles for Rate of Drift
0 1 2 3 4 5
Seconds of Arc
Trace of the Axis of Projectile or Locus of A
Curve of Rate of Lateral Drift
Trace of the Direction of Motion or Locus of A
Curve of Rate of Vertical Drift
Scale of Time in Seconds
0·0
0·1
0·2
0·3
0·4
0·5

Fig. 11.

Vertical Drift.

When neglecting the force of gravity we saw that the shot, in addition to drifting laterally, also became elevated above its initial direction of motion. The horizontal drift of X is due to the horizontal component of F_1X_0 (fig. 7), while the "vertical drift" of X, as distinguished from the vertical motion due to the weight, is due to the vertical component of F_1X_0. It is, of course, a very small quantity in practice; but in order to find its magnitude the two curves of horizontal and vertical rate of drift have been drawn in fig. 11, and on integrating them over the interval represented on the diagram, 0·6 second, the lateral drift is found to be 0·06 foot and the vertical drift 0·027 foot.

By dividing the time of flight of a projectile into periods of the precessional motion, and considering the velocity and deceleration as constant throughout each period but varying from period to period, it would be easy to trace out the complete history of the motions of the axis and of the path of flight for any given data regarding the shot and the resistances opposed to its motion.

In conclusion, I desire to express my cordial thanks to my friend and colleague Prof. W. Burnside, F.R.S., for the interest he has shown in the problem discussed, and my indebtedness to him for his valuable criticism.

[*Addendum, March* 3, 1909.—At the meeting of the Society when this paper was read, the question was raised whether the drift could possibly be due to the helical motion, since no trace of helical motion has been observed on the trajectory of a rifle bullet as recorded by the holes pierced in a series of screens. To settle this question, the curve of rate of drift for the first turn of the helical motion in fig. 9 was redrawn on a scale 10 times as great as in fig. 9, and was integrated. The linear horizontal drifts at the ends of the successive quarter periods are 0·27, 1·27, 2·26, and 2·85 inches, which, if plotted at equal intervals on a time base, will be found to correspond with a spiral of 0·4 inch radius. The deviation during this first turn of the spiral, it will be remembered, was taken as initially one degree, and is rapidly being decreased, hence 0·4 will be an excessive radius for the spiral path of a 12-inch shell. The cases on record where large projectiles have actually been seen travelling in spiral paths have probably been due to insufficient spin on the projectile, the driving band having slipped on the projectile or stripped in the gun.

A radius of 0·4 inch would be quite observable on the screen records of a small bullet, but the following elementary considerations of dimensions show us that the radius of the helix will be proportional to the diameter of the

projectile if the velocities and deviations of the axes are equal. The mass of the shot, $\propto d^3$; moment of inertia $\propto d^5$; resistance $\propto d^2$; tilting couple $\propto d^3$; hence the velocity of precession, which varies as the tilting couple $\div$ moment of inertia, $\propto d^{-2}$.

The curvature of the path due to the normal component of the resistance $\propto$ resistance $\div$ momentum of shot $\propto d^2/d^3 \propto d^{-1}$. Radius of the helix $\propto$ curvature of the path $\div$ velocity of precession $\propto d^{-1} \times d^2 \propto d$. The radius of the helix of a 0·3-inch bullet would therefore be of the order of $0{\cdot}4''/40 = 0{\cdot}01$ inch, which would not be detectable on the screen records.]

On the Graphical Determination of Fresnel's Integrals.

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The functions $\int_0^x \cos(\frac{1}{2}\pi x^2)\,dx$ and $\int_0^x \sin(\frac{1}{2}\pi x^2)\,dx$, known as Fresnel's integrals, have usually been evaluated by means of converging series, for instance those of Cauchy and Knochenhauer. The graphical method of integration, by the summation of the areas contained between given ordinates, the arc of the curve they cut off and the x-axis, is readily applicable in this case, the quantity $\cos\frac{1}{2}\pi x^2$ (or $\sin\frac{1}{2}\pi x^2$) being plotted as ordinate against x as abscissa. The area can then be determined between any given limits by Simpson's Rule. In practice, however, it is simpler and more accurate to apply Simpson's Rule directly to the calculated values of the ordinates, without plotting a curve.

The curve $y = \cos\frac{1}{2}\pi x^2$ has its zero values at $x = 1$, $3^{\frac{1}{2}}$, $5^{\frac{1}{2}}$, etc., and is thus marked off into a series of loops of steadily decreasing area. The simplest method of computation is to find the areas of these separate loops (the first being only half a loop) ranging, in values of x, from 0 to 1, 1 to $3^{\frac{1}{2}}$, $3^{\frac{1}{2}}$ to $5^{\frac{1}{2}}$, etc. The range for a particular loop will be termed its base-line.

In the subjoined tables the calculations have been made by dividing each base-line into 10 parts and calculating the ordinates corresponding to the points of division, *e.g.*, 0·1, 0·2, 0·3...0·9 for the first half-loop.

The area of any loop may then be written as kd, where d is the length of the base-line and k a factor depending upon the loop considered. The